Chaotic Dynamical Systems

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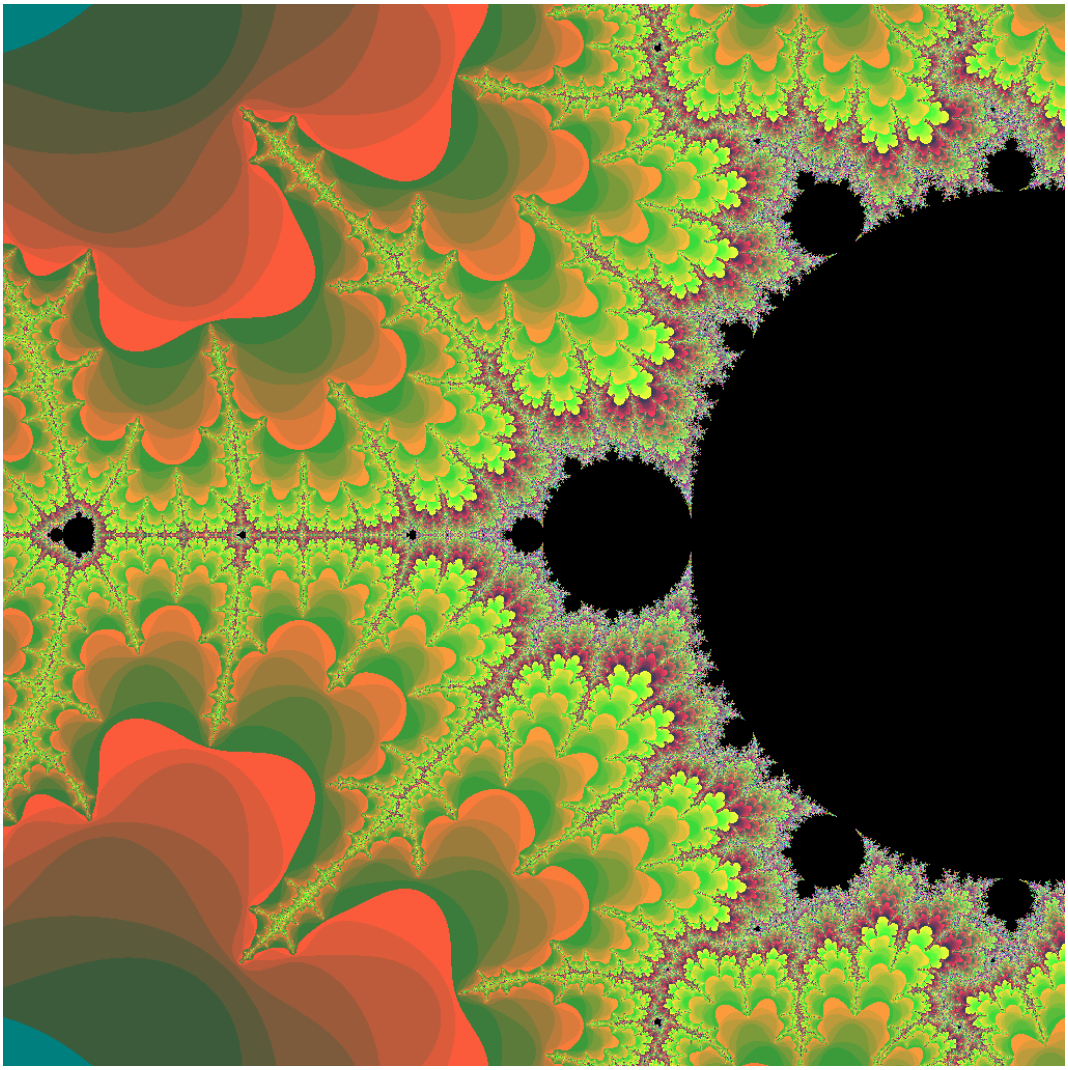
Office Hours: TBA
Office: TBA
Class Room: Online



- **Prerequisites:** Mathematical Analysis
- **Core/Elective:** Elective
- **Language:** English
- **Time:** Thursdays, 19:45 - 22:00
- **Place:** Online via Zoom
- **Schedule:**
  - First Lecture: 22nd of September
  - Last Lecture: 12th of Janunary

Course Description

This course aims to introduce students to fundamental concepts and recent research in the theory of chaotic dynamical systems. The focus will be on introducing students to the fundamentals of the field and giving them with a foundational understanding of the subject matter. This course examines the many definitions of Mathematical Chaos in a basic analytical manner.

Grading Policy

The grade will count the assessments using the following proportions:

- **20%** of your grade will be determined by writing exam.
- **80%** of your grade will be determined by 1 projects.

Module

- **Week 1:** Analysis of the dynamics (09/22/2022)
- **Week 2:** Dynamics of one-dimensional maps of both the interval and the circle (09/29/2022)
- **Week 3:** Recurrence and minimality (10/06/2022)
- **Week 4:** Elementary bifurcations (10/13/2022)
- **Week 5:** Sharkovskii's theorem (10/20/2022)
- **Week 6:** Li Yorke chaos and Scrambled sets (10/27/2022)
- **Week 7:** Transitivity and Devaney Chaos (11/03/2022)
- **Week 8:** Stronger forms of Transitivity (11/10/2022)
- **Week 9:** Symbolic Dynamics (11/17/2022)
- **Week 10:** Topological Entropy (11/24/2022)
- **Week 11:** Higher Dimensional Dynamics (12/01/2022)
- **Week 12:** Toral Automorphisms and Henon Map (12/08/2022)
- **Week 13:** Dynamical Billiards: Inner and Outer (12/15/2022)
- **Week 14:** Research Seminars of Students (01/12/2023)

Main References

1. Robert L. Devaney, *An Introduction To Chaotic Dynamical Systems*, Chapman and Hall/CRC, 3rd Edition, 2022.
2. Robert L. Devaney, *A First Course In Chaotic Dynamical Systems: Theory And Experiment* , Chapman and Hall/CRC, 2nd Edition, 2020.
3. Kathleen T. Alligood, Tim D. Sauer, James A. Yorke, *An Introduction to Dynamical Systems*, Springer; Corrected edition, 1996.

Research topics

- **Mathematical sciences:**

  1) Outer billiards around regular polygons
  2) The dynamics of a family of one-dimensional maps
  3) Inverse problems of symbolic dynamics
  4) The Role of Transitivity in Devaney's Definition of Chaos
  5) The Sharkovskii theorem
  6) Interactions between Dynamics, Arithmetics and Combinatorics

- **Applied sciences:**

  7) Artificial Intelligence and Chaos
  8) Dynamical Systems and Biology

Research References

1. A. Belov-Kanel, V. Timorin, F, Rukhovich, *Outer billiards around regular polygons*. Seminar of the Department of Geometry and Topology, Steklov Mathematical Institute of RAS, Moscow, Russia, 2022.

2. S. Bassein, *The Dynamics of a Family of One-Dimensional Maps* . The American Mathematical Monthly Vol. 105, No. 2 (Feb., 1998), pp. 118-130.

3. A. Kanel-Belov, G. Kondakov, I. Mitrofanov, *Inverse problems of symbolic dynamics* . arXiv:1104.5605

4. A. Crannell, *The Role of Transitivity in Devaney's Definition of Chaos*. The American Mathematical Monthly Vol. 102, No. 9 (Nov., 1995), pp. 788-793.

5. K. Burns , B. Hasselblatt, *The Sharkovsky Theorem: A Natural Direct Proof* . American Mathematical Monthly. 118 (3): 229–244.

6. V. Berthe, S. Ferenczi, L. Zamboni, *Interactions between Dynamics, Arithmetics and Combinatorics: the Good, the Bad, and the Ugly* . Algebraic and Topological Dynamics, American Mathematical Society; Contemporary Mathematics, 2005.

7. M. Sanjuan, *Artificial Intelligence, Chaos, Prediction and Understanding in Science*. arXiv:2003.01771

8. T. Azizi, B. Alali, G. Kerr, *Discrete Dynamical Systems: With Applications in Biology* . Book Publisher International, 2020.