Statistical Geometry

A. Kanel-Belov, Mehdi Golafshan

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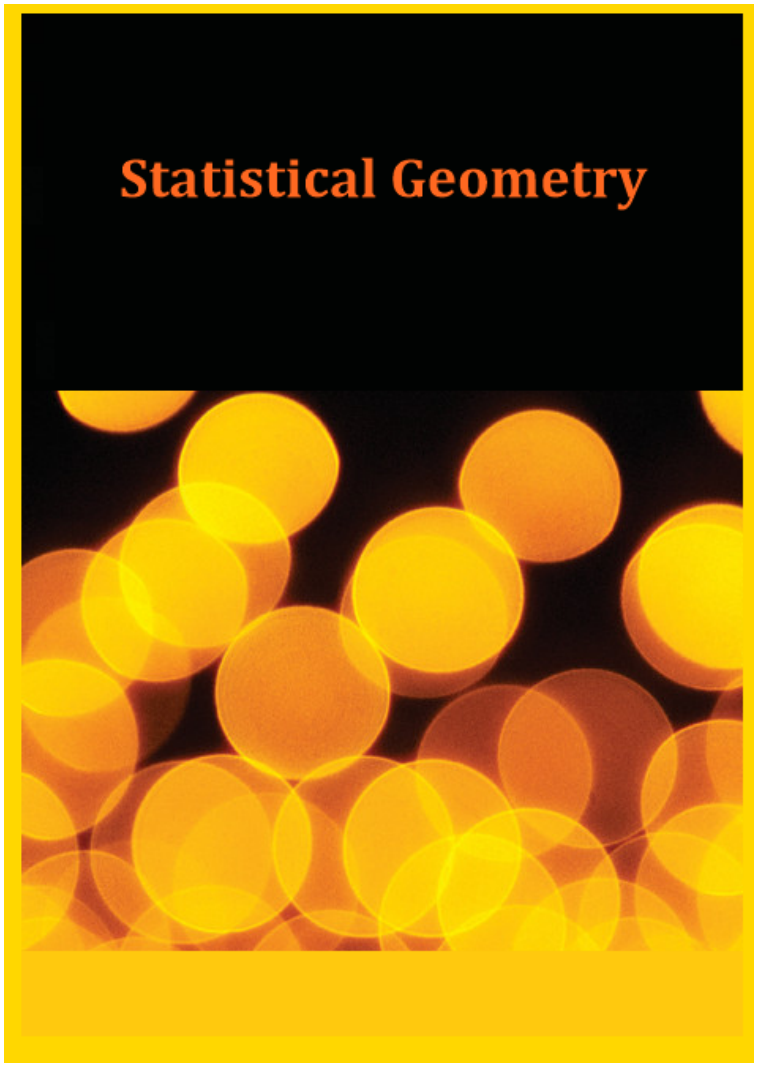
E-mail: mgolafshan@yandex.ru
Office Hours: Sat, 18-21
Office: golafshan.m.mehdi (my Skype)

Web: mipt.ru
Class Hours: Thu, 19-21
Class Room: Online



Course Description

$\mathcal{A}$. **Stochastic process:** A stochastic process or random process (see Stochastic calculation) or random function (see Probability) represents an evolution, discrete or in continuous time, of a random variable . This is involved in the classical calculation of probabilities, where it measures each possible result (or achievement) of a test.

This notion is generalized to several dimensions. An important special case, the Markov random field , is used in spatial analysis .

$\mathcal{B}$. **Stochastic geometry:** The stochastic geometry deals with the mathematical description and analysis of random geometric structures, such as dots or line segments, or more complicated quantities in space or plane. Random sets are an important basis .

An important application is the stereological acquisition of statements about spatial structures through the statistical analysis of linear and plane sections.

Various models of statistical mechanics (in particular, lattice models in two dimensions are considered here) such as percolation theory also result in random geometric structures that can be treated strictly mathematically using the Schramm-Löwner evolution method .

Course Outline

- **Markov Chains:** definition and examples, different types of chains (Ehrenfest, Weather, Repair Inventory, Two-Stage Markov, A Seven-State, Doubly Stochastic Birth and death, Renewal), Chapman–Kolmogorov equation, strong Markov property, monopoly examples
- **Random Walks:** definition and examples, symmetric reflecting random walk on the line, random walks on graphs, random walk of a knight on a chess board, reflecting random walk
- **Poisson Processes:** exponential distribution, Poisson distribution, definition of Poisson processes, basic properties of Poisson processes, nonhomogeneous Poisson processes, compound Poisson processes and examples, thinning and superposition, a Poisson race, conditioning
- **Renewal Processes:** definition of renewal processes, laws of large numbers and renewal processes, alternating renewal processes
- **Continuous Time Markov Chains:** definition and examples, jump rate, Yule process, Kolmogorov's backward and forward equation, branching processes, stationary distribution, exit distribution, exit times, queueing networks, infinite state space
- **Martingales:** conditional expectation, martingale, supermartingale, submartingale, predictable process, exponential martingale gambling strategies, stopping times, predictable process, wald's equation, left-continuous random walk)
- **Brownian Motions:** Wiener process, standard Brownian motion, diffusion processes, geometric Brownian motion, Brownian motion in two or three dimensions, semi-Markov processes
- **Foundations of Stochastic Geometry:** Buffon's needle, Cauchy-Crofton formula, Bertrand's paradox, integral geometry in the plane
- **Random Sets:** definition of a random closed set, examples of random Closed sets (Singleton, Half-line, random interval, random triangle, random ball, random line, random set in finite space, random variables associated with random closed sets (Indicator, Norm, Measure of random set, Capacity functionals
- **Discrete Stochastic Geometry:** random polytopes, random polyhedral sets, random mosaics, Poisson–Voronoi mosaics

Prerequisites/Corequisites

Probability, Geometry, Differential Equation

Main References

1. R. Durrett, *Essentials of Stochastic Processes*, Springer International Publishing Switzerland; 2016.
2. Sh. Ross, *Stochastic Processes* , Wiley; 2nd edition, 1995.
3. P. Jones and P. Smith, *Stochastic Processes An Introduction* , Chapman and Hall/CRC Press, 2017.
4. D. Coupier, *Stochastic Geometry, Modern Research Frontiers* , Springer Nature Switzerland AG 2019.
5. L. Santalo, *Integral Geometry and Geometric Probability* , Addison-Wesley Publishing Company, 1976.
6. I. Molchanov, *Theory of Random Sets* , Springer-Verlag London Ltd; 2017.
7. D. Klain and G. Rotta, *Introduction to Geometric Probability* , Cambridge University Press 1997.
8. A. Kanel-Belov, S. Malev, R. Yavich, M. Golafshan, *ABOUT RANDOM SPLITTING OF THE PLANE* , Crimean Autumn Mathematical School-Symposium on Spectral and Evolutionary Problems.
9. M. Golafshan, A. Kanel-Belov, *Using statistical geometry for partition of hyper planes* , Conference: Lomonosov 2021.

Grading Policy

- Assignments; (80%)
- Projects. (20%)

Assignments

5 series of homework: each section one homework include of one or two problems

Project

Based on materials of class